On the Complex Nature of Higher Order Modes in Lossless Nonreciprocal Transversely Magnetized Waveguides

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Abstract—**Propagation of nonreciprocal modes in transversely magnetized lossless nonreciprocal waveguides is analyzed. It is shown that purely evanescent modes cannot exist if the wave propagation is nonreciprocal. As a consequence of this all the modes which do not support a net average power flux must be complex. These modes are called here pseudo-evanescent complex modes. The meaning of the cutoff frequency concept of such pseudo-evenescent complex modes is also discussed.**

I. Introduction

Complex modes in reciprocal lossless inhomogeneous waveguides [1]–[4], as well as in reciprocal homogeneously and anisotropically filled waveguides [5] have been reported in the literature. In all these cases, a plane wave with a z dependence of the fields e^{-jkz} was assumed, where $k = \beta - j\alpha$ is the complex propagation constant, β is the phase constant and α is the attenuation constant. It has also been reported that four values of the propagation constant k, $-k$, k^* and $-k^*$ are present. More recently complex modes in nonreciprocal lossless waveguides have been investigated [6].

In the present letter, wave propagation in transversely magnetized lossless nonreciprocal waveguides is analyzed. It is shown that all the modes which do not carry a net average power flux, become complex when the wave propagation is nonreciprocal. These modes are called here *pseudo-evanescent complex modes*

II. Theory

Let a waveguide be inhomogeneously filled with lossless gyrotropic materials (see Fig. 1), having the following properties:

$$\overline{\epsilon_i}(-\boldsymbol{H}_{o,i}) = \overline{\epsilon_i}^*(\boldsymbol{H}_{o,i}) \tag{1}$$

$$\overline{\boldsymbol{\mu}_i}(-\boldsymbol{H}_{o,i}) = \overline{\boldsymbol{\mu}_i}^*(\boldsymbol{H}_{o,i}). \tag{2}$$

In the following, we will denote $\{\boldsymbol{H}_{o,i}, k\}$ a mode in the waveguide magnetized with the dc fields $\boldsymbol{H}_{o,i}$, having complex propagation constant k. We say that this mode is a *nonreciprocal* mode (with respect to the propagation constant) if there is no other mode $\{\boldsymbol{H}_{o,i}, -k\}$ with opposite propagation constant.

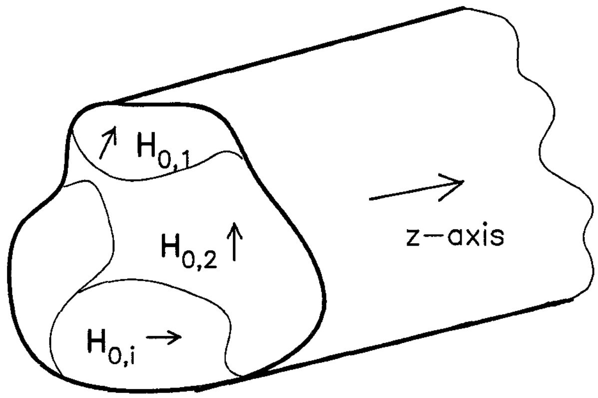

Fig. 1. A general lossless waveguide inhomogeneously filled with transversely magnetized gyrotropic media.

If Maxwell's equations are applied to the $\{\boldsymbol{H}_{o,i}, k\}$ mode of the waveguide, the following equations are satisfied in each homogeneous subsection:

$$-jk\boldsymbol{a}_z \times \boldsymbol{E}_i + \nabla_t \times \boldsymbol{E}_i = -j\omega\overline{\boldsymbol{\mu}}_i \cdot \boldsymbol{H}_i \tag{3}$$

$$-jk\boldsymbol{a}_z \times \boldsymbol{H}_i + \nabla_t \times \boldsymbol{H}_i = j\omega\overline{\epsilon}_i \cdot \boldsymbol{E}_i, \tag{4}$$

where $\boldsymbol{E}(x,y)$ and $\boldsymbol{H}(x,y)$ are the radiofrequency fields in the ith homogeneous subsection, associated with the mode $\{\boldsymbol{H}_{o,i}, k\}$.

Taking the complex conjugate and after some calculations, the same equations are obtained for the radiofrequency fields $(\boldsymbol{E}_i^*, -\boldsymbol{H}_i^*)$, provided k changes to $-k^*$ and the constitutive tensor are replaced by by $\overline{\epsilon}_i^*$, $\overline{\boldsymbol{\mu}}_i^*$ (i.e., if the dc magnetizing field $\boldsymbol{H}_{o,i}$ is replaced by $-\boldsymbol{H}_{o,i}$). Since the new boundary conditions imposed by the change of $\boldsymbol{H}_{o,i}$ to $-\boldsymbol{H}_{o,i}$ are also satisfied by $(\boldsymbol{E}_i^*, -\boldsymbol{H}_i^*)$, we conclude that *if* $\{\boldsymbol{H}_{o,i}, k\}$ *is a mode of the waveguide*, $\{-\boldsymbol{H}_{o,i}, -k^*\}$ *represents also a mode of the guide.*

Let us now consider the latter mode $\{-\boldsymbol{H}_{o,i}, -k^*\}$. After a reflection in the transverse $(x - y)$ plane, and taking into account the electromagnetic field properties with spatial reflection [7], this mode becomes a $\{\boldsymbol{H}_{o,i}, k^*\}$ mode, provided that the dc bias field $\boldsymbol{H}_{o,i}$ is contained in the transverse $(x-y)$ plane. Therefore, in transversely magnetized waveguides, the existence of the $\{\boldsymbol{H}_{o,i}, k\}$ mode, implies the existence of the $\{\boldsymbol{H}_{o,i}, k^*\}$ mode.

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TABLE I
COMPLEX PROPAGATION CONSTANT IN mm^{-1} IN A FERRITE-LOADED PARALLEL-PALATE WAVEGUIDE[1] WITH $h_f = 2$ mm, $h_d = 8$ mm, $\epsilon_d/\epsilon_o = 1$, $\epsilon_f = 12.6$, $H_0 = 500$ Oe, $f_r = 10$ GHz

Mode	$4\pi M_s = 0$	$4\pi M_s = 100$	$4\pi M_s = 300$	$4\pi M_s = 1000$	$4\pi M_s = 2000$
1	0 $\pm j0.0849$	0.0028 $\pm j0.0876$	0.0075 $\pm j0.0957$	0.0249 $\pm j0.1407$	0.0396 $\pm j0.2368$
2	0 $\pm j0.5021$	0.0027 $\pm j0.5025$	0.0082 $\pm j0.5035$	0.0322 $\pm j0.5112$	0.1093 $\pm j0.5469$
3	0 $\pm j0.8539$	0.0014 $\pm j0.8539$	0.0044 $\pm j0.8540$	0.0169 $\pm j0.8531$	0.0495 $\pm j0.8361$
4	0 $\pm j1.1953$	$0.24 \cdot 10^{-3}$ $\pm j1.1953$	$0.74 \cdot 10^{-3}$ $\pm j1.1955$	0.0030 $\pm j1.1962$	0.0088 $\pm j1.928$

[1] See Fig. 2.

Notice that purely evanescent modes, $\{\boldsymbol{H}_{o,i}, -j\alpha\}$, can not be nonreciprocal, since in this case, the reciprocal mode $\{\boldsymbol{H}_{o,i}, j\alpha\}$ must also exist. On the other hand, it is a well-known fact that all modes that do not carry a net average power flux must be either evanescent or complex. Thus, since the existence of nonreciprocal purely evanescent modes in transversely magnetized waveguides is impossible, it turns out that *all nonreciprocal modes that do not support a net average power flux must be complex in this type of waveguide.*

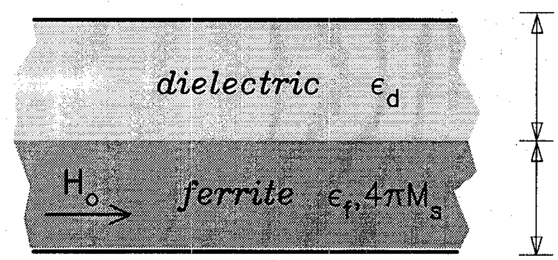

Fig. 2. Ferrite-loaded parallel-plate waveguide transversely magnetized.

III. NUMERICAL EXAMPLES

In this section, we will analyze the dispersion characteristics of nonreciprocal TE modes in a ferrite-loaded parallell plate waveguide. The method of analysis will be the same as that presented in [8], and it will not be developed here.

The effects of the nonreciprocity on the propagation constant of the TE modes are shown in Table I. This table shows the variation in the propagation constant of the first four TE modes with respect to the magnitude of the internal magnetization of the ferrite slab (which is varied from 0 to 2.000 Gauss). The operation frequency (fr) is chosen in such a way that the four modes considered were purely evanescent in the reciprocal case ($M_s = 0$). It is clearly observed how the presence of nonreciprocity makes the evanescent reciprocal modes ($M_s = 0$) turn into *nonreciprocal pseudo-evanescent complex modes* with the aforementioned properties.

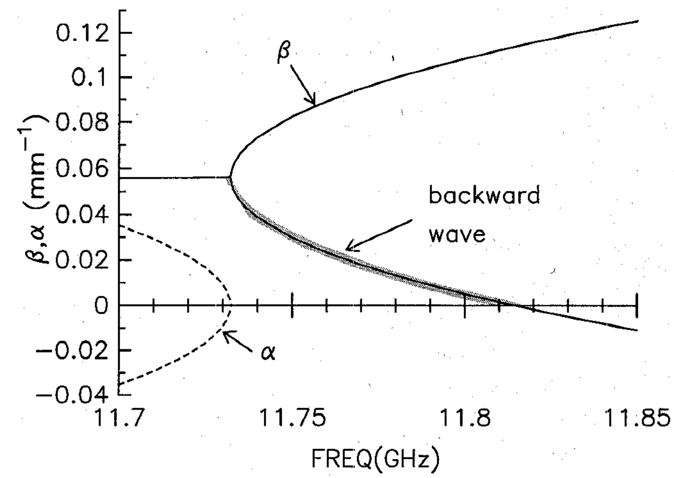

Fig. 3. Phase constant β: (———) and attenuation factor α: (– – – –) in mm^{-1} of the first TE mode in a ferrite-loaded parallel-plate waveguide (see Fig. 2), with $h_f = 2$ mm,$h_d = 8$ mm, $\epsilon_d/\epsilon_0 = 1$, $\epsilon_f/\epsilon_0 = 12.6$, $4\pi M_s = 2000$ G, $H_0 = 500$ Oe.

Fig. 3 shows the dispersion of the attenuation and phase constant of the first TE mode of the above waveguide. Reading this figure from the right to the left, it can be seen how the transition from propagating to pseudo-evanescent complex modes takes place at a nonzero value of the phase constant, and after the phase constant of one of the propagating modes passes through zero. This behavior has been found in all the analyzed pseudo-evanescent modes, and it is in agreement with the well known fact that complex modes can appear as a combination of a forward and a backward wave [9]. This fact suggests that the cutoff frequency of pseudo-evanescent modes can be defined as the frequency at which the complex mode appears. This cutoff frequency does not coincide with the frequency at which the phase constant of the propagating modes passes through zero.

IV. CONCLUSION

It has been shown that nonreciprocal purely evanescent modes can not appear in transversely magnetized nonreciprocal waveguides. *Pseudo-evanescent nonreciprocal complex modes take the place of such modes in these waveguides.* Numerical investigations suggest that pseudo-evanescent complex modes always appear as a combination of a backward and a forward wave, after the phase constant of one of these waves (the backward one) passes through zero. Thus, the cutoff frequency of pseudo-evanescent complex modes does not coincide with the frequency at which the phase constant passes through zero.

Nonreciprocal pseudo-evanescent complex modes are a type of complex modes. They are complex modes because they have a complex propagation constant and do not carry a net average pover flux. Nevertheless, these modes also preserve certain properties of the evanescent modes, such as the existence of a cutoff frequency. The propagation constant is purely real for frequencies above the the cutoff frequency but it turns out to be complex (with nonzero real and imaginary parts) below the cutoff frequency. Pseudo-evanescent complex modes turn into purely evanescent modes as the cause of the nonreciprocity dissapears (for instance when the external magnetic field is removed).

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